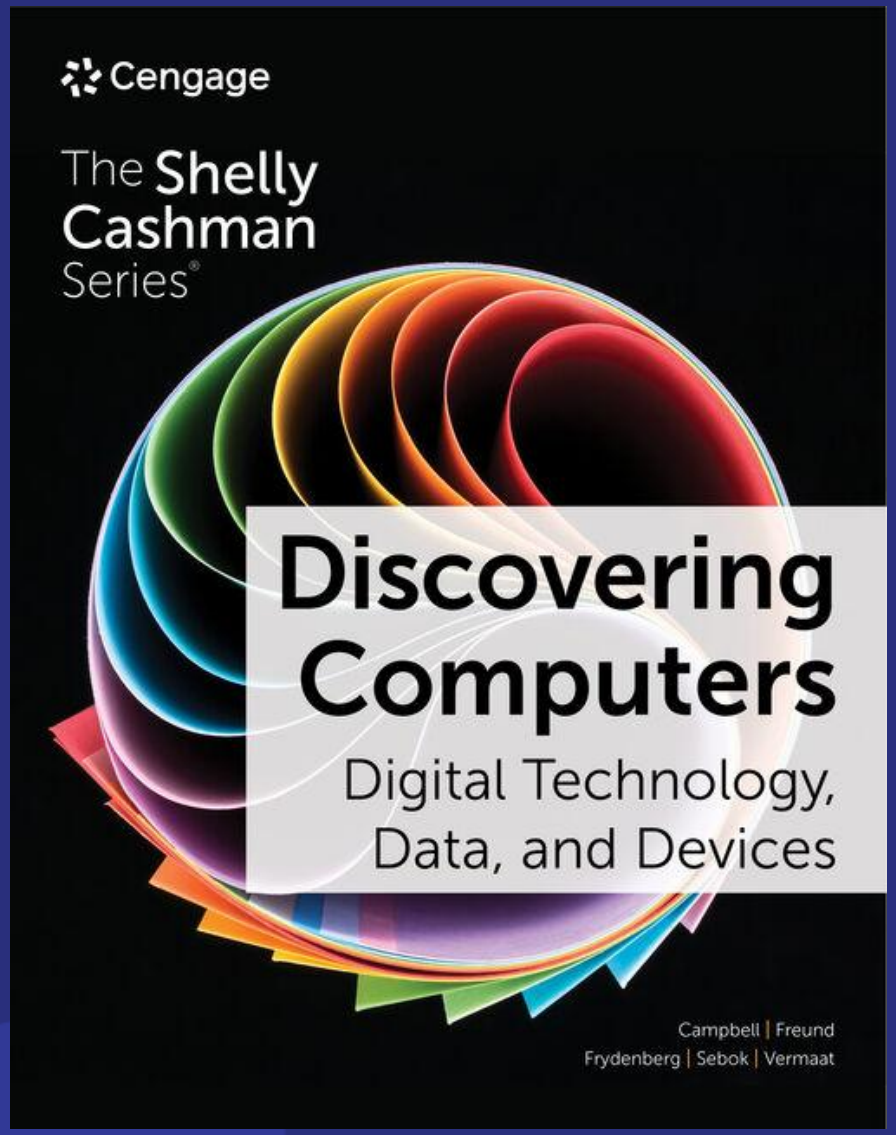


CS 110 – Introduction To Computing

Module 4: Data Representation

Cengage

Introduction

- We use computer to process the data and get the desired output.
- The data input can be in the form of alphabets, digits, symbols, audio, video, magnetic cards, finger prints, etc.
- Since digital computer can only understand 0 and 1, the data must be represented in the computer in 0s and 1s.

Introduction

- Since data will be stored in using a number system, it is important to study number systems that are used in the computer.
- The number systems discussed in this lecture are— (1) Decimal number system, (2) Binary number system, (3) Octal number system, and (4) Hexadecimal number system.

Number System

- A number System is a writing system for expressing numbers, i.e., a mathematical notation for representing numbers of a given set.
- A number system in base r uses unique symbols for r digits.
- One or more digits are combined to get a number.

Number Systems

- The base of the number decides the valid digits that are used to make a number.
- In a number, the position of digit starts from the right-hand side of the number.
- The rightmost digit has position 0, the next digit on its left has position 1, and so on.

Number Systems

- The digits of a number have two kinds of values—
- Face value, and
- Position value.
- The **face value** of a digit is the digit located at that position. For example, in decimal number 52, face value at position 0 is 2 and face value at position 1 is 5.

Number Systems

- The **position value** of a digit is (base ^{position}). For example, in decimal number 52, the position value of digit 2 is 10^0 and the position value of digit 5 is 10^1. Decimal numbers have a base of 10.

- The **number** is calculated as the sum of, face value * base ^{position}, of each of the digits. For decimal number 52, the number is $5*10^1 + 2*10^0 = 50 + 2 = 52$.

Number Systems of interest

- Decimal Number System—Base 10
- Binary Number System—Base 2
- Octal Number System—Base 8
- Hexadecimal Number System—Base 16

A number in a particular base is written as $(number)_{base\ of\ number}$. For example, $(23)_{10}$ means that the number 23 is a decimal number, and $(345)_8$ shows that 345 is an octal number.

Decimal Number System

- I t consists of 10 digits—0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- All numbers in this number system are represented as combination of digits 0–9. For example, 34, 5965 and 867321.

Decimal Numbers

Position:	3	2	1	0	.	–1	–2	–3
Position Value:	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
Quantity:	1000	100	10	1		1/10	1/100	1/1000

Octal Number System

- The octal number system consists of eight digits—0 to 7.
- All octal numbers are represented using these eight digits. For example, 273, 103, 2375, etc.

Octal Number System

Position:	3	2	1	0	.	−1	−2	−3
Position Value:	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
Quantity:	512	64	8	1		1/8	1/64	1/512

Hexadecimal Number System

- The hexadecimal number system consists of sixteen digits—0 to 9, A, B, C, D, E, F, where (A is for 10, B is for 11, C-12, D-13, E-14, F-15).
- All hexadecimal numbers are represented using these 16 digits.
- For example, 3FA, 87B, 113, etc.

Hexadecimal Number System

Position:	3	2	1	0	.	−1	−2	−3
Position Value:	16^3	16^2	16^1	16^0		16^{-1}	16^{-2}	16^{-3}
Quantity:	4096	256	16	1		1/16	1/256	1/4096

Conversion from Decimal to Binary, Octal, Hexadecimal

- A decimal number has two parts—integer part and fraction part.
- Eg. In the number 23.078, 23 is the integer part and 0.078 is the fraction part.

- The method used for the conversion of the integer part of a decimal number is different from the one used for the fraction part.

Conversion from Decimal to Binary (Convert 25 from Base 10 to Base 2)

1. Make a table as shown . Write the number in centre and to Base on the left side.

to Base	Number (Quotient)	Remainder
2	25	

Converting 25 from base 10 to Base 2

2. Divide the number with to Base. After each division, write the remainder on right-side column and quotient in the next line in the middle column. Continue dividing till the quotient is 0.

to Base	Number (Quotient)	Remainder
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

Converting 25 from base 10 to base 2

3. Write the digits in remainder column starting from downwards to upwards

The binary equivalent of number $(25)_{10}$ is $(11001)_2$.

to Base	Number (Quotient)	Remainder
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

Try

- Convert the following numbers from base 10 to base 2, 8 16

a) 53
b) 68
c) 29

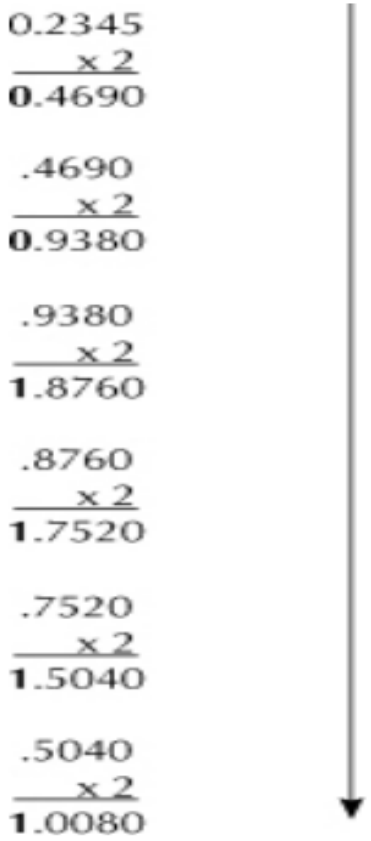
Converting Decimal Fraction to other bases

- A fractional number is a number less than 1.
- **To convert a decimal fraction to—**
- binary-multiply by 2,
- octal-multiply by 8, and,
- Hexadecimal-multiply by 16.

Converting fractions (Steps)

- 1. Multiply the fractional number with the to Base, to get a resulting number.
- 2. The resulting number has two parts, non-fractional part and fractional part.
- 3. Record the non-fractional part of the resulting number.
- 4. Repeat the above steps at least four times.
- 5. Write the digits in the non-fractional part starting from upwards to downwards.

Convert 0.2345 from Base 10 to Base 2.



Convert 0.2345 from Base 10 to Base 2.

- The binary equivalent of $(0.2345)_{10}$ is $(0.001111)_2$

Try

- Convert the following decimal fractions to base 2,8 and 16

a) 0.865

b) 0.455

Converting Decimal Integer. Fraction to other bases

- The steps for conversion of a decimal fraction to any other base are—
- 1. Convert decimal integer part to the desired base following the steps given earlier
- 2. Convert decimal fraction part to the desired base following the steps shown earlier
- 3. The integer and fraction part in the desired base is combined

Try

- Convert 34.4674 from Base 10 to Base 2.
- Convert 34.4674 from Base 10 to Base 8.

Conversion of Binary, Octal, Hexadecimal to Decimal

- Numbers have two parts—integer part and fraction part.
- The method used for the conversion of integer part and fraction part of binary, octal or hexadecimal number to decimal number is the same;
- multiplication operation is used for the conversion.

Conversion of Binary, Octal, Hexadecimal to Decimal (Steps)

1. Find the sum of the **Face Value * (from Base)** $^{\textbf{position}}$ for each digit in the number.

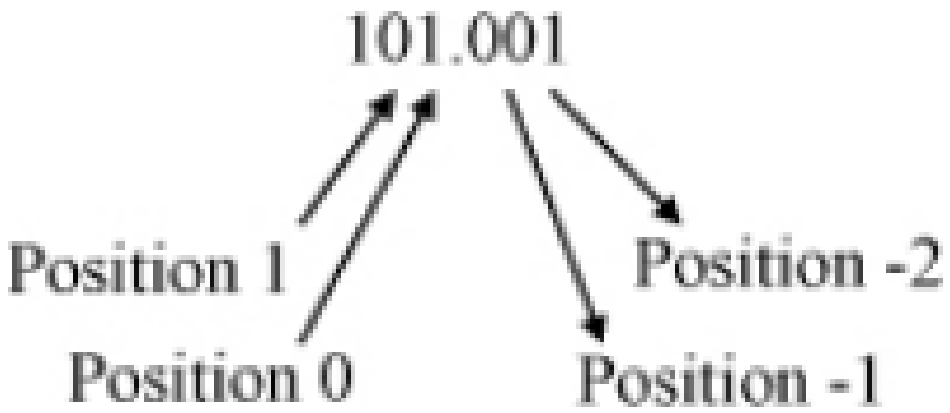
a) In a non-fractional number, the rightmost digit has position 0 and the position increases as we go towards the left.

Conversion of Binary, Octal, Hexadecimal to Decimal (Steps)

b. In a fractional number, the first digit to the left of decimal point has position 0 and the position increases as we go towards the left.

- The first digit to the right of the decimal point has position –1 and it decreases as we go towards the right (–2, –3, etc.)

Conversion of Binary, Octal, Hexadecimal to Decimal (Steps)



Conversion of Binary, Octal, Hexadecimal to Decimal (example)

- Convert 1011 from Base 2 to Base 10.
- $1011 = 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0$

- Convert C15 from Base 16 to Base 10.
- $C15 = C*16^2 + 1*16^1 + 5*16^0$
- $= 12*256 + 1*16 + 5*1$

Conversion of Binary, Octal, Hexadecimal to Decimal (example)

- Convert .1101 from Base 2 to Base 10.
- $.1101 = 1*2^{-1} + 1*2^{-2} + 0*2^{-3} + 1*2^{-4}$

- Convert .15 from Base 16 to Base 10.
- $.15 = 1*16^{-1} + 5*16^{-2}$
- $= 1/16 + 5/256 = 21/256 = 0.82$

Conversion of Binary, Octal, Hexadecimal to Decimal (example)

- Convert 1011.1001 from Base 2 to Base 10.
- $1011.1001 = 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0 + 1*2^{-1} + 0*2^{-2} + 0*2^{-3} + 1*2^{-4}$
- $= 8 + 0 + 2 + 1 + 1/2 + 0 + 0 + 1/16$
- $= 11 + 9/16 = 11.5625$

- Convert 4D.21 from Base 16 to Base 10.

Conversion of Binary to Octal, Hexadecimal

- A binary number can be converted into octal or hexadecimal number using a shortcut method.
- The shortcut method is based on the following information —

i. An octal digit from 0 to 7 can be represented as a combination of 3 bits, since $2^3 = 8$.

ii. A hexadecimal digit from 0 to 15 can be represented as a combination of 4 bits, since $2^4 = 16$.

The Steps for Binary to Octal Conversion

- 1. Partition the binary number in groups of three bits, starting from the right-most side.
- 2. For each group of three bits, find its octal number.
- 3. The result is the number formed by the combination of the octal numbers.

Binary to Octal Conversion (Example)

- Convert the binary number 1110101100110 to octal.
- Solution
- 1. Partition binary number in groups of three bits, starting from the right-most side.
- 1 1 1 0 1 0 1 1 0 0 1 1 0

Binary to Octal Conversion (Example)

- 2. For each group find its octal number.
- 1 110 101 100 110
- 1 6 5 4 6

- 3. The octal number is 16546.

- Convert the binary number 1100111100111 to Octal

Steps for Binary to Hexadecimal

- 1. Partition the binary number in groups of four bits, starting from the right-most side.
- 2. For each group of four bits, find its hexadecimal number.
- 3. The result is the number formed by the combination of the hexadecimal numbers.

Binary to Hexadecimal (Example)

- Convert the binary number 1110101100110 to hexadecimal
- Solution
- 1. Partition binary number in groups of four bits, starting from the right-most side.

1 1101 0110 0110

Binary to Hexadecimal (Example)

- 2. For each group find its hexadecimal number.

1	1 1 0 1	0 1 1 0	0 1 1 0
1	D	6	6

- 3. The hexadecimal number is 1D66.

Conversion of Octal, Hexadecimal to Binary

- The conversion of a number from octal and hexadecimal to binary uses the inverse of the steps defined for the conversion of binary to octal and hexadecimal.

Steps for Hexadecimal to Binary Conversion

- 1. Convert each hexadecimal number into a four-digit binary number moving from right to left.
- 2. The result is the number formed by the combination of all the bits.

Conversion of Hexadecimal to Binary

- 1. Given number is 2BA3
- 2. Convert each hexadecimal digit into four digit binary number.
- 2 B A 3
- 0010 1011 1010 0011
- 3. Combine all the bits to get the result 0010101110100011.

Steps for Octal to Binary Conversion

- 1. Convert each octal number into a three-digit binary number moving from right to left.
- 2. The result is the number formed by the combination of all the bits.

Conversion of Octal to Binary

- Convert the octal number 473 to binary.
- Solution
- 1. Given number is 473
- 2. Convert each octal digit into three digit binary number.
- 4 7 3
- 100 111 011
- 3. Combine all the bits to get the result 100111011.